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WAVE ANALOG OF THE COMMON DEPTH POINT METHOD

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Obtained is a new solution to the problem of defining a small velocity inhomogeneity from a diffracted wave field recorded by a multielement system of receivers with successive excitation of identical sources. The effect of accumulation due to redefinition of the source data allows implementation of a CDP approach to suppress multiply scattered wave interference in visualizing random velocity inhomogeneities. The effectiveness of the constructed algorithm is verified by numerical modeling.

At the present time one of the main methods of processing seismic exploration data is the common depth point (CDP) method [7-9]. This method uses systems of observations of multiple overlapping. Summation of seismic traces from CDP traveltime curves together with the procedure of analyzing the velocity spectrum leads to amplification of the useful signal and suppression of multiply reflected wave interference. These advantages of the CDP method let us reconstruct smooth, almost horizontal interfaces of the inhomogeneities of the investigated stratified media.

However, the CDP method in essence does not use dynamic characteristics of reflected waves. Therefore, it does not always reconstruct sharp irregular velocity variations which lead to diffraction scattering of sounding signals. Because of this it is interesting to create a method of processing which would preserve the advantages of the CDP method and moreover, considering the wave nature of the scattering process, would reconstruct both the smooth interfaces and the sharp inhomogeneities of the investigated medium. With this purpose this work discusses for the wave equation the problem of defining a weak finite velocity inhomogeneity from the field which is scattered at this inhomogeneity and recorded by a multielement areal system of receivers with successive switching of identical sources.

The inverse problem of scattering of a vertically incident plane wave at a weak velocity inhomogeneity was discussed in [3-5], which derived theorems of existence and uniqueness. It was shown that a scattered plane wave can define the spectrum of the sought velocity function along some contour in a complex plane. However, part of this contour lies not on the real axis, but assumes complex values; therefore in evaluating the inverse Fourier transform the integral over this area of the contour has poor noise-resistance which exponentially deteriorates with depth. Varying the angle of incidence of the wave, we can, of course, obtain the spectrum of the sought function on the real axis, but in this case we are unable to implement the procedure of accumulation of the sought function, which is so effective in the CDP method.

In the present work we have obtained a new, rigorous solution of the problem of defining a weak velocity inhomogeneity from a diffracted wave field recorded by a multielement system of sources and receivers. It is shown that this solution lets us not only calculate the velocity variation in a stable manner, but also implement the procedure of accumulation. The result obtained is a natural form of the solution to the problem for recording systems of multiple overlapping. The effect of accumulation of the sought function due to the redefinition of the source data opens the way for the use of a method similar to the CDP method in order to suppress multiply scattered wave interference in the visualization of random (not weak) velocity inhomogeneities. We will give some numerical examples of the visualization of such inhomogeneities which illustrate the proposed algorithm. The author believes that this method can be useful for practical processing of seismic exploration data with visualization of deep-seated and complicatedly structured inhomogeneities of the investigated media.

The algorithm proposed was first described by the author in 1980 [6].

PROBLEM FORMULATION

We will now look at the inverse problem of diffraction of sound waves propagating in an unbounded medium with velocity c at an inhomogeneous inclusion which occupies a limited region F (Fig. 1). The wave field $u = u(r, \rho_0, t)$ is excited at the time $t = 0$ by a point pulse source located on the plane $z = 0$ at the point $\rho_0 = (x_0, y_0)$ and is described by the equation

$$\Delta u - \frac{1}{c^2}[1 + a(\mathbf{r})]\,u_{tt} = \delta(\mathbf{r} - \rho_0)\,\delta(t). \tag{1}$$

We will pose the problem of defining the function $a(\mathbf{r})$ from a scattered field $u(\rho, \rho_0, t)$, known on the plane $z = 0$ for different positions of the source and receiver. We note that these data are redundant; this will be used for the accumulation of the sought function.

Let

$$u(\mathbf{r}, \rho_0, t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} u(\mathbf{r}, \rho_0, t) \tag{2}$$

be the Fourier representation of the wave field. Each of the harmonics of the wave field $u(r, \rho_0, \omega)$ satisfies the Helmholtz equation, which with consideration of the condition of radiation is equivalent, as we know, to the integral equation

$$u(\mathbf{r}, \mathbf{r}_0, k) = G_0(k|\mathbf{r} - \mathbf{r}_0|) - k^2\int d\mathbf{r}' u(\mathbf{r}, \mathbf{r}', k)a(\mathbf{r}')G_0(k|\mathbf{r}_0 - \mathbf{r}'|). \tag{3}$$

here $G_0(kr) = -\exp(ikr)/(4\pi r)$ is the Green function for a homogeneous space, and $k = \omega/c$ is the wave number. It is also easy to be convinced that the following equation is valid:

$$u(\mathbf{r}, \mathbf{r}_0, k) = G_0(k|\mathbf{r} - \mathbf{r}_0|) - k^2\int d\mathbf{r}' G_0(k|\mathbf{r} - \mathbf{r}'|)a(\mathbf{r}')G_0(k|\mathbf{r}' - \mathbf{r}_0|) + \\ + k^4\int d\mathbf{r}'d\mathbf{r}'' G_0(k|\mathbf{r} - \mathbf{r}''|)a(\mathbf{r}'')u(\mathbf{r}'', \mathbf{r}', k)a(\mathbf{r}')G_0(k|\mathbf{r}' - \mathbf{r}_0|). \tag{4}$$

Actually, substituting expression (4) into the Helmholtz equation and using (3), we obtain an identity.

The Green function $G_0(k|\mathbf{r} - \mathbf{r}_0|)$ describes the propagation of a wave in a homogeneous space from the point $\mathbf{r}_0$ to the point $\mathbf{r}$; therefore the first term in the right side of relationship (4) corresponds to a direct wave. The second term of this relationship describes the propagation of a wave from the point $\mathbf{r}_0$ to the point $\mathbf{r}'$, scattering at the point $\mathbf{r}'$ with amplitude $a(\mathbf{r}')$, and then the passage of the wave from the point $\mathbf{r}'$ to the observation point $\mathbf{r}$. This term describes a singly scattered wave. Finally, the last term corresponds to waves of higher multiples of scattering. We will consider, as in the CDP method, that singly scattered waves are informative waves, whereas multiply scattered waves are wave interference. We must determine and by using the data redundancy accumulate the sought function $a(\mathbf{r})$ with respect to singly scattered waves, assuming that the procedures of accumulating and sorting velocities will permit suppression of multiply scattered waves. This assumption can evidently be confirmed only empirically. Rigorous estimates of the contribution of multiple waves will be presented below only for weak velocity inhomogeneities $a(\mathbf{r})$, i.e., in the Rayleigh-Born approximation.

Thus, we have the problem of defining the function $a(\mathbf{r})$ from the equation

$$v(\rho, \rho_0, k) = -k^2\int d\mathbf{r}' G_0(k|\rho - \mathbf{r}'|)a(\mathbf{r}')G_0(k|\mathbf{r}' - \rho_0|) \tag{5}$$

according to the function $v(\rho, \rho_0, k) = u(\rho, \rho_0, k) - G_0(k|\rho - \rho_0|)$. We carry out Fourier

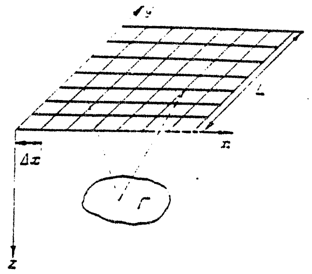

Fig. 1. Scattering of sound wave at inhomogeneous inclusion Γ; ρ_0 and ρ are points of the position of the source and receiver on the plane z = 0.

transformation of equation (5) with respect to variables ρ and ρ_0. Having used the formul from [1]

$$\int d\rho \exp\left(i\varkappa\rho \pm i|k|\sqrt{\rho^2+z^2}\right)(\rho^2+z^2)^{-1/2} = \pm\frac{2\pi i}{\sqrt{k^2-\varkappa^2}}\exp\left(\pm i|z|\sqrt{k^2-\varkappa^2}\right), \qquad (6)$$

we obtain

$$v(\varkappa, \varkappa_0, k) = \frac{1}{4}\frac{k^2}{\sqrt{k^2-\varkappa^2}\sqrt{k^2-\varkappa_0^2}}\int dz' \exp\left[\pm iz'\left(\sqrt{k^2-\varkappa^2} + \sqrt{k^2-\varkappa_0^2}\right)\right] a(\varkappa+\varkappa_0, z'). \qquad (7)$$

Here the branches of the roots should be selected such that $\sqrt{k^2-\varkappa^2} = \pm i\sqrt{\varkappa^2-k^2}$ for $|\varkappa| < \kappa$, and the upper and lower signs correspond to the cases of $k > 0$ and $k < 0$. We note that the integral with respect to the variable z' in the right side of relationship (7) has the form of a Fourier integral. If k is varied from $-\infty$ to $+\infty$, then the parameter $p = \pm\left(\sqrt{k^2-\varkappa^2} + \sqrt{k^2-\varkappa_0^2}\right)$ describes in a complex plane some contour C which consists of the intervals $\left(-\infty, -\sqrt{|\varkappa^2-\varkappa_0^2|}\right)$, $\left(\sqrt{|\varkappa^2-\varkappa_0^2|}, \infty\right)$ of the real axis and semicircumference of radius $\sqrt{|\varkappa^2-\varkappa_0^2|}$. which lies in the upper halfplane. Thus, relationship (7) allows us for $p \in C$ to express the Fourier image of $a(\kappa + \kappa_0, p)$ of the sought function $a(\mathbf{r})$ in terms of the known function $v(\rho, \rho_0, k)$. For this we look at the inverse Fourier transform:

$$a(\varkappa+\varkappa_0, z) = \int \frac{dp}{2\pi} e^{-ipz} a(\varkappa+\varkappa_0, p). \qquad (8)$$

We will now deform the path of integration to the contour C. Having replaced the variable $p = \pm\left(\sqrt{k^2-\varkappa^2} + \sqrt{k^2-\varkappa_0^2}\right)$ for the area of the contour which lies in the halfplanes Re p > 0 and Re p < 0 respectively, and having used relationship (7), we obtain

$$a(\varkappa+\varkappa_0, z) = \int dk\,|k|^{-1}\left(\sqrt{k^2-\varkappa^2} + \sqrt{k^2-\varkappa_0^2}\right) \times \exp\left[\mp iz\left(\sqrt{k^2-\varkappa^2} + \sqrt{k^2-\varkappa_0^2}\right)\right] \cdot v(\varkappa, \varkappa_0, k). \qquad (9)$$

From this for each fixed κ_0 we have

$$a(\mathbf{r}) = \int \frac{d\varkappa}{(2\pi)^2} dk |k|^{-1} \left(\sqrt{k^2 - \varkappa^2} + \sqrt{k^2 - \varkappa_0^2} \right) \exp\left[i(\varkappa + \varkappa_0)\rho \mp \right.$$
$$\left. \mp iz\left(\sqrt{k^2 - \varkappa^2} + \sqrt{k^2 - \varkappa_0^2} \right) \right] v(\varkappa, \varkappa_0, k). \qquad (10)$$

We note that integral (10) has poor noise-resistance. For example, for the value $\kappa_0 = 0$ (corresponding to a vertically incident plane wave) the branches of the roots in the exponential cofactor are such that if $\kappa > |k|$ we multiply $v(\kappa, 0, k)$ by the exponent $\exp(z\sqrt{\varkappa^2 - k^2})$ which increases with the increase in z. The presence of such inhomogeneous waves leads to the fact that small errors in the source data can greatly alter the result of the calculations.

The presence in expression (10) of the free parameter κ_0 corresponds to the redundancy of source data and lets us implement the procedure of accumulation of the sought function. Having adopted the task of constructing an algorithm for defining the function $a(\mathbf{r})$ from the entire aggregate of data $v(\rho, \rho_0, k)$, we note that because of the reciprocity theorem $v(\rho, \rho_0, k) = v(\rho, \rho_0, k)$. Therefore, both the variables ρ and ρ_0 and the corresponding spectral vectors κ and κ_0 should enter this algorithm symmetrically. These considerations remind us that to accumulate the sought function we should integrate both sides of expression (10) with respect to the variable κ_0. However, in this case we obtain divergent integrals. The physical nature of these divergences is related to the circumstance that each point source scatters the waves at the inhomogeneity Γ which have finite energy, and the continual number of sources corresponds to infinitely large energy, and this is what leads to divergent integrals. For adequate physical formulation of the problem we note that in areal systems of observation the excitation and recording of waves is accomplished at the nodes of a grid of points located in the aperture $L \times L$ with interval Δx between them (see Fig. 1). The value of Δx defines the resolution of $\lambda \sim 1/K$ and the greatest spatial frequency $K = 2\pi/\Delta x$, which corresponds to this system of observation. More precise mathematical analysis, carried out below, shows that if we discard inhomogeneous waves, multiply the integrand of (10) by the factor

$$m = \left[1 + (\varkappa + \varkappa_0)^2 / \left(\sqrt{k^2 - \varkappa^2} + \sqrt{k^2 - \varkappa_0^2} \right)\right]^{1/2} \qquad (11)$$

and integrate both sides of relationship (10) with respect to all physically different vectors κ_0, we obtain the sought operator of focusing

$$\beta_K(\mathbf{r}) = 8\pi L^2 \int_0^K dk \int \frac{d\varkappa}{(2\pi)^2} \int \frac{d\varkappa_0}{(2\pi)^2} \theta(k^2 - \varkappa^2)\, \theta(k^2 - \varkappa_0^2) \times$$
$$\times \Phi(\varkappa, \varkappa_0, k, \mathbf{r})\, v(\varkappa, \varkappa_0, k) + \text{c. c.}, \qquad (12)$$

where $\theta(x)$ is the Heaviside function,

$$\Phi \equiv k^{-1}\left(k^2 + \varkappa\varkappa_0 + \sqrt{k^2 - \varkappa^2}, \sqrt{k^2 - \varkappa_0^2}\right)^{1/2} \cdot \exp\left[i(\varkappa + \varkappa_0)\rho - \right.$$
$$\left. - iz\left(\sqrt{k^2 - \varkappa^2} + \sqrt{k^2 - \varkappa_0^2}\right)\right], \qquad (13)$$

and c.c. designates the complex conjugation of the preceding term, here and henceforth. We emphasize that we have not substantiated the transition from expression (10) to focusing operator (12). Therefore, focusing operator (12) requires detailed mathematical study, which we will carry out in the next section.

SUBSTANTIATION OF ALGORITHM

In this section we will formulate and prove a properly conditioned mathematical statement which lets us substantiate algorithm (12) proposed above.

We will define the functions:

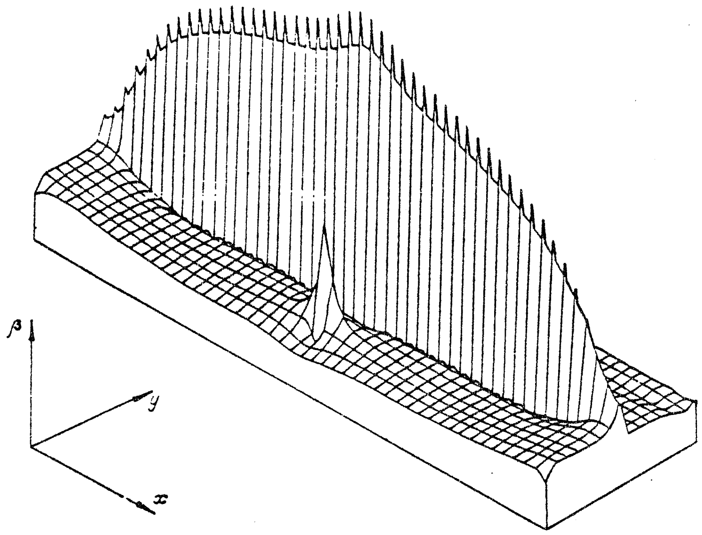

Fig. 2. Relief of function $\beta_K(x, y)$ for scattering point and reflecting plane.

$$\Delta_K(\mathbf{r}) = \frac{2}{K^2}\int_0^K dk \int \frac{d\varkappa}{(2\pi)^2}\int \frac{d\varkappa_0}{(2\pi)^2}\theta(k^2-\varkappa^2)\,\theta(k^2-\varkappa_0^2)\times$$
$$\times[\Phi(\varkappa, \varkappa_0, k, \mathbf{r}) + \Phi^*(\varkappa, \varkappa_0, k, \mathbf{r})], \tag{14}$$

$$a_K(\mathbf{r}) = \int d\mathbf{r}'\Delta_K(\mathbf{r}-\mathbf{r}')\,a(\mathbf{r}'). \tag{15}$$

Lemma $\lim\limits_{K\to\infty} a_K(\mathbf{r}) = a(\mathbf{r})$ for the random finite function $a(\mathbf{r}) \in C^\infty$.

Proof. In integral (14) we will replace the variables $\varkappa = k\mathbf{x}$, $\varkappa_0 = k\mathbf{x}_0$, $k = K\xi$. Then

$$\Delta_K(\mathbf{r}) = K^2\cdot f(K\rho, Kz). \tag{16}$$

Here $\mathbf{r} = (\rho, z)$, and the function

$$f(\mathbf{r}) = 2\int_0^1 d\xi\xi^4 \int \frac{d\mathbf{x}}{(2\pi)^2}\int \frac{d\mathbf{x}_0}{(2\pi)^2}\theta(1-x^2)\,\theta(1-x_0^2)\times$$
$$\times[\Phi(\mathbf{x}, \mathbf{x}_0, 1, \xi\rho, \xi z) + \text{c.c.}]. \tag{17}$$

It is shown in the Appendix that the spectrum

$$F(\varkappa, p) = \int d\rho dz \exp(-i\varkappa\rho - ipz) f(\rho, z) \tag{18}$$

of the function $f(\mathbf{r})$ has the properties:

$$|F(\mathbf{s})| \leqslant 1,\ F(\mathbf{s}) = 0 \text{ for } |\mathbf{s}| > 1,\ F(\mathbf{s}) = 1 + 0(s^2) \text{ for } |\mathbf{s}| \leqslant 1. \tag{19}$$

Here $s = (\kappa, p)$. Having used the theorem on the Fourier transformation of the convolution of functions, we obtain

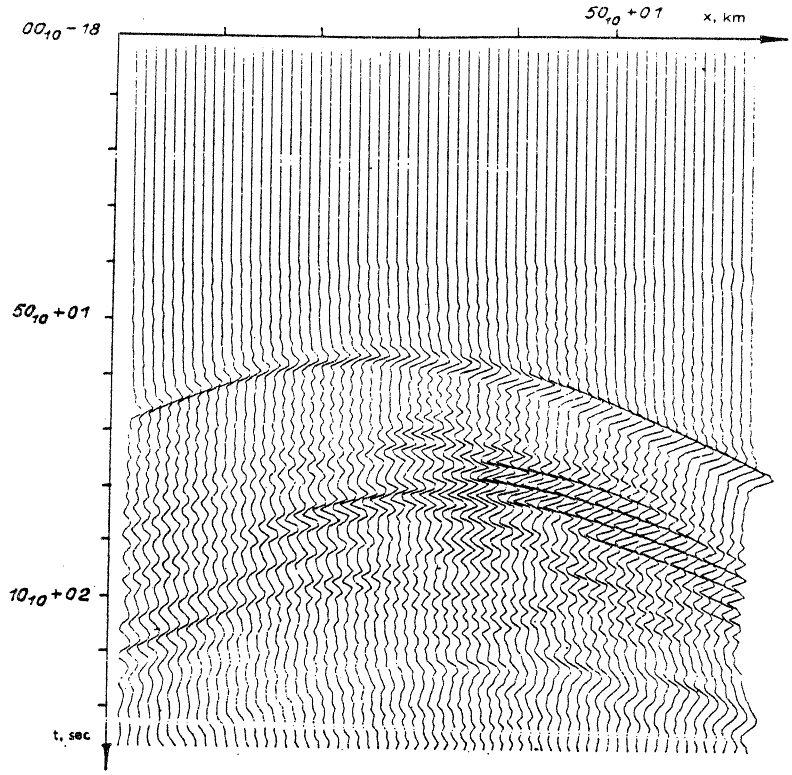

Fig. 3. Montage of traces of wave field scattered on cylindrical inclusion with elliptical cross section. The source is located at the point $x_0 = 6.4$ km, $c_1 = 1$ km/sec, $c_2 = 0.9$ km/sec.

$$a_K(\mathbf{r}) = \int \frac{d\mathbf{s}}{(2\pi)^3} e^{i\mathbf{s}\mathbf{r}} F\left(\frac{\mathbf{s}}{K}\right) a(\mathbf{s}). \tag{20}$$

From the properties (19) of the spectrum $F(\mathbf{s})$ it follows that the spectrum of the function $a_K(\mathbf{r})$ coincides with the spectrum of the function $a(\mathbf{r})$ for frequencies $|\mathbf{s}| \leqslant K$. Going to the limit $K \to \infty$ under the sign of the integral (20), we obtain the proof of the lemma.

The function $\Delta_K(\mathbf{r})$ is an approximation of the Dirac δ function. From relationships (19) it follows that the function $a_k(\mathbf{r})$ is obtained from the function $a(\mathbf{r})$ as a result of the local averaging of the latter with respect to volume with dimensions $\lambda \sim 1/K$.

Theorem. Let $M = \sup|a(\mathbf{r})|$ be the contrast of the velocity inhomogeneity, and l is the diameter of the area Γ. Then for the function

$$\alpha_K(\mathbf{r}) = \frac{2^{3/2}}{K^2}\int_0^K dk \int \frac{d\varkappa_0}{(2\pi)^2}\frac{d\varkappa_0}{(2\pi)^2}\,\theta(k^2-\varkappa^2)\,\theta(k^2-\varkappa_0^2)\times$$
$$\times[\Phi(\varkappa, \varkappa_0, k, \mathbf{r})\,v(\varkappa, \varkappa_0, k) + \text{c.c.}] \tag{21}$$

we have the bound: $|\alpha_K(\mathbf{r}) - a_K(\mathbf{r})| \leqslant M^2K^5l^5$.

Proof. We will now calculate the Fourier transformation of relationship (4) with respect to the variables ρ and ρ_0. Having used formula (6), we obtain:
where

$$v_0(\varkappa, \varkappa_0, k) = \frac{1}{4} \frac{k^2}{\sqrt{k^2-\varkappa^2}\sqrt{k^2-\varkappa_0^2}} \int d\mathbf{r}' \exp\Big[-i(\varkappa+\varkappa_0)\rho' \pm \pm iz'\Big(\sqrt{k^2-\varkappa^2}+\sqrt{k^2-\varkappa_0^2}\Big)\Big] a(\mathbf{r}'), \tag{22}$$

$$\delta v(\varkappa, \varkappa_0, k) = \frac{1}{4} \frac{k^2}{\sqrt{k^2-\varkappa^2}\sqrt{k^2-\varkappa_0^2}} \int d\mathbf{r}' d\mathbf{r}'' \exp\Big(-i\varkappa\rho'' - i\varkappa_0\rho' \pm iz''\sqrt{k^2-\varkappa^2} \pm iz'\sqrt{k^2-\varkappa_0^2}\Big) a(\mathbf{r}'') u(\mathbf{r}'', \mathbf{r}', k) a(\mathbf{r}'). \tag{23}$$

We will act on the spectrum of the diffracted field $v(\kappa, \kappa_0, k)$ with integral operator (21). As a result of the action of this operator on $v_0(\kappa, \kappa_0, k)$ we obtain the function $a_K(\mathbf{r})$. We estimate the result of the effect of operator (21) on $\delta v(\kappa, \kappa_0, k)$. For $\mathbf{r}, \mathbf{r}_0 \in \Gamma$ and $|k| \leqslant K$ the wave field satisfies the inequality.

$$|u(\mathbf{r}, \mathbf{r}_0, k)| \leqslant \frac{1}{4\pi}\left(\frac{1}{|\mathbf{r}-\mathbf{r}_0|} + \frac{\varepsilon}{1-\varepsilon}\cdot\frac{1}{l}\right), \tag{24}$$

where $\varepsilon = MK^2\ell^2$. Having used this inequality and relationship (23), we obtain that $|\alpha_K(\mathbf{r}) - a_K(\mathbf{r})| \leqslant \varepsilon^2 \cdot Kl$.

We note that the function $\alpha_K(\mathbf{r})$, which we calculate from the diffracted field, turns out to be close to the local averaging $a_K(\mathbf{r})$ of the sought function $a(\mathbf{r})$, if the parameter $\varepsilon \ll 1$. The smallness of this parameter is a condition of the applicability of the Rayleigh-Born approximation, which amounts to the fact that the velocity inhomogeneity slightly distorts the incident and scattered waves. The condition $\varepsilon = MK^2\ell^2 \ll 1$ sets a limit on the resolution $\lambda \sim 1/K$ of the definition of the function $a(\mathbf{r})$ and the magnitude of the contrast of the inclusion M.

WAVE ANALOG OF THE CDP METHOD

The solution we obtained in the previous sections for the model problem of defining weak velocity inhomogeneities can form the basis for the method of visualization of random (not weak) inhomogeneities similar to the CDP method. Actually, this solution essentially uses the observation system of multiple overlapping. Averaging the calculated function with respect to these redundant data not only increases its statistical reliability, but also lets us implement the ideology of the CDP method with consideration of the wave nature of the sounding process. In this section we will present a discussion analogous to the standard explanations of the practical effectiveness of the CDP method and some numerical examples which illustrate the efficiency of the proposed algorithm for visualizing strong inhomogeneities.

The scattered wave field $u(\rho, \rho_0, k)$ will be represented in the form of the superposition of the useful singly scattered waves and multiply scattered wave interference. The structure of these waves is analogous to that of the corresponding terms in relationship (4) with some effective propagation velocities. As a result of the action of focusing operator (12) on singly scattered waves we obtain

$$\beta_K(\mathbf{r}) = N \cdot a_K(\mathbf{r}), \tag{25}$$

where $N = L^2/\Delta x^2$ is the number of points on the aperture which characterizes the degree of redefinition of the source data. Thus, as a result of the action of focusing operator (12) on singly scattered waves we obtain the accumulation of local averaging $a_K(\mathbf{r})$ of the sought

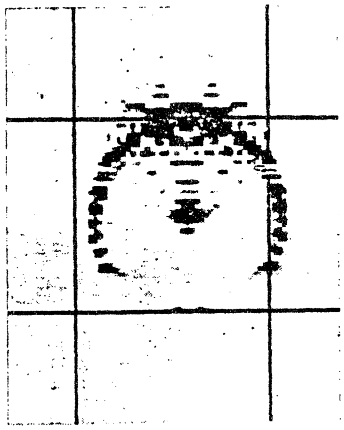

Fig. 4.

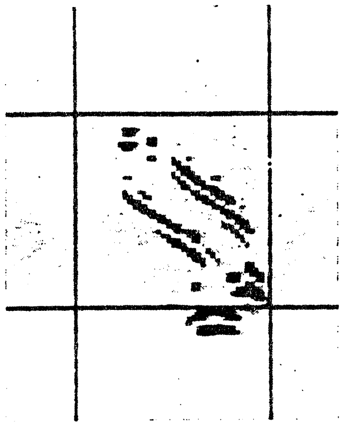

Fig. 5.

Fig. 4. Result of reconstruction of cylindrical inclusion with circular cross section, c_1 = 1 km/sec; c_2 = 2 km/sec.

Fig. 5. Result of reconstruction of cylindrical inclusion with elliptical cross section; c_1 = 1 km/sec, c_2 = 0.9 km/sec.

function $a(\mathbf{r})$, proportional to the degree of redefinition of the source data. This accumulation occurs due to cophasal summation of the useful signal, since operator (12) considers the geometry of single scattering and compensates for the oscillating factors in expression (22) which correspond to the temporal lags in the propagation of incident and scattered waves. For multiply scattered waves there is no such compensation and it seems natural that the accumulation of these terms occurs, generally speaking, in some random manner. This lets us assume that upon an increase in the repetition of overlapping N the proposed algorithm, together with the procedure of analysis of the velocity spectrum [9], will let us effectively visualize complicatedly structured inhomogeneous media. The latter requires further experimental confirmation.

In conclusion, we will describe some numerical experiments which illustrate the described algorithm. For simplicity all calculations were carried out for the plane case. The velocity of the enclosing medium c was assumed equal to 1 km/sec. The distance Δx between the points on the x axis at which the sources were placed and the scattered field was calculated is equal to 0.1 km. The number of such points on the aperture N = 64, Ω = Kc = 30 rad/sec.

a) Gently sloping and smooth velocity interfaces are typical structures for seismic exploration. The simplest model of such boundaries is a reflecting plane. However, visualization of sharp local velocity inhomogeneities is a very difficult task for the CDP method. We considered a medium containing a scattering point located in the middle of the aperture with abscissa x_p = 3.2 km at a depth y_p = 1 km and a reflecting plane at the depth y_{pl} = 2 km. The wave field scattered at the point was modeled by the expression

$$v_p(x, x_0, k) = -k^2 G_0\left(k\sqrt{(x-x_p)^2+y_p^2}\right) G_0\left(k\sqrt{(x_0-x_p)^2+y_p^2}\right) \tag{26}$$

(compare with (4)); here $G_0(k\rho) = i/4 \cdot H_0^{(1)}(k\rho)$ is the Green function of the Helmholtz equation on the plane, $H_0^{(1)}(z)$ is the Hankel function; x_0, x are the abscissas of the source and receiver, located on the line y = 0; (x_p, y_p) are the coordinates of the scattering point. The field reflected from the plane was calculated by the formula

$$v_{pl}(x, x_0, k) = G_0\left(k\sqrt{(x-x_0)^2+4y_{pl}^2}\right). \tag{27}$$

The total field scattered by this system consists of four terms which correspond to different ray trajectories: source-point-receiver, source-plane-receiver, source-point-plane-receiver, and source-plane-point-receiver. These fields were calculated for all 64 positions of the source and then processed. The processing results are illustrated in Fig. 2, where the position of both the scattering point and the reflecting plane is clearly seen.

b) The following examples illustrate the possibilities of the algorithm in reconstructing finite velocity inhomogeneities, cylindrical inclusions with circular (with D = 2 km) and elliptical cross sections. The centers of these inclusions are located at the point with coordinates $x = 3.2$ km, $y = 3$ km, the diameter of the major axis of the ellipse D = 2 km, and the ratio of the major to minor axes D/d = 3. The scattered wave field was calculated according to the programs of Voronin [2]. We emphasize that for these models the parameter $\varepsilon \sim 10^3$, i.e., much greater than one.

Figure 3 presents as an example a montage of traces of the wave field excited at the point $x_0 = 6.4$ km and scattered by a cylindrical inclusion with an elliptical cross section and velocity $c_2 = 0.9$ km/sec.

Figure 4 illustrates the result of reconstructing the cylindrical inclusion with a circular cross section. We emphasize that along with the horizontally positioned elements of the velocity interface we also reconstructed well the vertically situated elements of the surface. The reconstruction of these surface elements evidently cannot be explained by ray theories, and it is conditioned by purely wave effects. We also note the correct reconstruction of the lower elements of the surface which for the normal ray according to kinematic considerations should be placed in the center of the circle. However, the use of the redundant multiaspect information and consideration of the wave nature of the process of scattering let us obtain their correct position.

Figure 5 illustrates the results of reconstructing the cylindrical inclusion with an elliptical cross section.

We note that although in the examples of section b) the parameter $\varepsilon \gg 1$, i.e., the formal conditions of applicability of the theory we developed are not fulfilled, the author believes that qualitatively the result of visualization of these inhomogeneities is very good.

APPENDIX

We will investigate the spectrum $F(\kappa, p)$ of the function $f(\rho, z)$. Having used relationships (17) and (18) and the identity

$$\delta[\xi(x + x_0) - \varkappa] = \int dq\delta(\xi x - \varkappa/2 - q)\,\delta(q - \varkappa/2 + \xi x_0), \tag{28}$$

we obtain

$$\begin{gathered} F(2\varkappa, 2p) = \frac{1}{\pi}\sqrt{1 + 4\frac{\varkappa^2}{p^2}}\int_0^1 d\xi\xi \int dq\theta[\xi^2 - (q + \varkappa)^2] \times \\ \times\,\theta[\xi^2 - (q - \varkappa)^2]\left(\sqrt{\xi^2 - (q + \varkappa)^2} + \sqrt{\xi^2 - (q - \varkappa)^2}\right)/\left(\sqrt{\xi^2 - (q + \varkappa)^2}\,\times \right. \\ \left.\times\sqrt{\xi^2 - (q - \varkappa)^2}\right)\left[\delta\left(2p + \sqrt{\xi^2 - (q + \varkappa)^2} + \sqrt{\xi^2 - (q - \varkappa)^2}\right) + \right. \\ \left. + \,\delta\left(2p - \sqrt{\xi^2 - (q + \varkappa)^2} - \sqrt{\xi^2 - (q - \varkappa)^2}\right)\right]. \end{gathered} \tag{29}$$

From relationship (29) we can see that $F(\kappa, p) = F(\kappa, -p)$. For definiteness we will look at the case $p > 0$; then the first of the δ functions in the right side of formula (28) makes a zero contribution. To calculate the remaining integral, we note that the equation

$$\gamma = 2p - \sqrt{\xi^2 - (q + \varkappa)^2} - \sqrt{\xi^2 - (q - \varkappa)^2} = 0 \tag{30}$$

has the real roots

$$\hat{q} = \pm p\sqrt{\xi^2 - p^2 - \varkappa^2}\,/\,\sqrt{p^2 + \varkappa^2\cos^2\varphi} \tag{31}$$

for $\kappa^2 + p^2 \leq \xi^2$; here ϕ is the angle between the vectors q and κ.

Since

$$\left.\frac{\partial\gamma}{\partial q}\right|_{q=\hat{q}} = 2\frac{\hat{q}}{p}(p^2+\varkappa^2\cos^2\varphi)[\xi^2-(\hat{q}+\varkappa)^2]^{-1/2}\cdot[\xi^2-(\hat{q}-\varkappa)^2]^{-1/2}, \qquad (32)$$

then

$$F(2\varkappa, 2p) = \frac{1}{\pi}\sqrt{1+4\frac{\varkappa^2}{p^2}}\int_0^1 d\xi\cdot\xi\theta(\xi^2-p^2-\varkappa^2)\int_0^{2\pi} d\varphi\theta[\xi^2-(\varkappa+\hat{q})^2]\times$$
$$\times\theta[\xi^2-(\varkappa-\hat{q})^2]\cdot p^2\cdot(p^2+\varkappa^2\cos^2\varphi)^{-1}. \qquad (33)$$

From this it is easy to obtain the sought relationships (19).

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